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Quantify distinguishability in Robotics

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Abstract. Through a Dynamical Systems approach to robotics, we introduce a measure of distinguishability between different behaviours of a robot, in an uncertain context, thanks to interval analysis.

Keywords. mobile robotics, dynamical systems, discrimination, interval analysis

1. Introduction

Our long-term goal is (i) to do predictible and efficient environment recognition, with a simple low-cost robot, and (ii) to know before the experiment if such an aim is reachable given the environmental constraints. To achieve this goal, we make two hypotheses: first, the states of the robot form a continuum, but not all of them can be distinguished by the robot itself. Secondly, the recognition capabilities of a robot depend strongly on the density of those distinguishable states among the continuous state-space. Furthermore, we think that environment recognition mustn't be defined alone, but comes side-by-side with a main interaction in the environment (e.g. obstacle avoidance, of homing). Admitting those hypotheses, this article sets the basis for a quantified and predictive approach to robotics, in order to know before any experiment what one can expect or not from a robot as regards its capacity to distinguish one of its own behaviour from another, in an uncertain context.

To do so, we adapt mathematical tools belonging to Dynamical Systems Theory, and mix them with interval analysis so as to deal with uncertainty. More precisely, our proposition to fulfill this aim is to quantify how many states a robot can distinguish, per volume unit in the state-space. In following works, we will consider improving that number with an appropriate feedback.

Section 2 is devoted to stating some definitions, especially (n, ε)-separation, and focuses on a toy problem. Section 3 extends those considerations to an experimental context, while section 5 summarizes the results and gives some insight of future works.

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2. Theoretical state-counting

2.1. Background

From a macroscopic point of view, the robot is a system on which forces apply, openned to a flow of energy. To give a kinematic description of its state, we set a state-space and each point in it represents a unique state of the system. But we're also interested in a dynamical description : which transformation in the phase-space does the robot undergo during its activity ? In this paper we adopt a dynamical systems approach (see [1]) and will consider a discrete time and model the state of a robot as a point in a multidimensional state-space X. The interaction of the robot with its environment is viewed as an application $T : X \to X$ that $\forall x \in X$ associates $T(x)$.

2.2. Noise

Our system is mainly driven by deterministic forces, but we also need to take fluctuations into account, since the interaction itself can be partly random, or because of the data acquisition process. For example, we could take a purely deterministic application T from X to X, and $\vec{\omega}$ a random variable taking its values in X. The application S would act on $\vec{x}$ as shown by Eq.(1). Then, our aim would be to find the probability density function (pdf) of $S^n(\vec{x_0})$, given the pdf of $\vec{x_0}$, where we hold S^n is the n-th iteration of S. In the literature, tools such as stochastic differential equations or Fokker-Planck equations could do the job. Instead, we propose in this article *interval analysis* (see [3]) as a first approach to model uncertainty, because we need to catch a glimpse of the overall behaviour of the system without being hindered by the intractable calculus that quickly comes with those frameworks. From now we will consider the interval $[x]$ instead of x to model the intrinsic randomness of T, as well as $[\omega]$ instead of ω to model the uncertainty due to the measurement process. In the rest of this article, we will study the trajectory resulting from the repeated action of S on some initial condition $[\vec{x_0}]$, as shown by equation 2.

$$\vec{x} \xrightarrow{S} T(\vec{x}) + \vec{\omega} \tag{1}$$

$$[\vec{x}] \xrightarrow{S} T([\vec{x}]) + [\omega] \tag{2}$$

where a *trajectory* is the set $\{\vec{x}, S(\vec{x}), \ldots, S^n(\vec{x})\}$. We will need the following notations: $\underline{\omega}$ and $\overline{\omega}$ are respectively the lower and upper bounds of ω, hence $\omega = [\underline{\omega}, \overline{\omega}]$. Besides, we call $W([\omega])$ the *width* of interval $[\omega]$, i.e. the length $\overline{\omega} - \underline{\omega}$.

2.3. Tools from Dynamical Systems Theory

As a consequence of hypotheses made in section 1, we look for a way to count the number of "distinguishable" states in a given subspace of the state-space. To define the notion of "distinguishability", we refer to some definitions given by R. Bowen on the way to redefining topological entropy (see [2]). Let's start introducing a two distances on X, with T an application from X to X:

Definition 1

$$d(x,y) = |x-y|, \text{ i.e. the euclidean norm on } X \tag{3}$$

$$d_{n,T}(x,y) = \max_{i<n} d(T^i(x), T^i(y)) \tag{4}$$

an illustration of which is given by Fig.1(a). When there is no ambiguity concerning the application, we will simply put d_n instead of $d_{n,T}$. Now we introduce the main definition:

Definition 2 *let K be a subset of $E \subset X$, n an integer and $\varepsilon \in \mathbb{R},\ \varepsilon > 0$. We say that $K \subset E$ is an (n,ε)-separated set with respect to T if, for every $(x,y) \in K$, we have : $x \neq y \Rightarrow d_n(x,y) > \varepsilon$.*

Given an arbitrary finite precision ε, two points are thus considered distinct if the application T makes them more distant than ε after n iterations. Knowing if a set of points is separated, we now count the maximum number of points satisfying that constraint, in a limited domain of the state-space:

Definition 3 *in the conditions of Definition 2, $s_n(\varepsilon, E)$ is the largest cardinality of any (n,ε)-separated set K contained in E.*

2.4. Example

Let $T : x \rightarrow \alpha x$ be a deterministic scalar application defined on $\mathbb{R}$, that models the dynamics of the robot in state space. Though it looks much too simple an example, it will prove to be useful since such exponential behaviour arise frequently in robotics. First, we look for a set K of (n,ε)-separated points in some subset E of $\mathbb{R}$, admitting that it has a predefinite geometry governed by a small number of parameters. In this example, as shown by Fig. 1(b), $E = [a,b]$ and the points of K are separated by a constant distance d along the real line. The question we ask is how to choose d such that the points of K are (n,ε)-separated, and what is the maximal value of d ? As mentionned in 2.2, the answer we propose is given in an interval analysis framework, a basic tool of which is *set inversion*. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a function, and V a subset of $\mathbb{R}^m$. Set inversion allows to characterize $U = f^{-1}(V) = \{x \in \mathbb{R}^n | f(x) \in V\}$, thanks to subpavings U^- and U^+ such that $U^- \subset U \subset U^+$. In the present case, we use the algorithm SIVIA (see [3] for more precisions) to perform this characterization and yield U^- and U^+ with an arbitrary resolution. Applying this method to find d such that points of K are (n,ε)-separated, we get a set of solutions as shown by Fig. 1(c), where red indicates guaranteed solutions, blue stands for rejected points, and yellow shows areas where the resolution was insufficient to conclude. According to the results, d must exceed approximately $2/3$ so that all points of $K \subset E$ are (n,ε)-separated by T.

Now we add some measurement noise to our model, represented here by an interval ω, and note $S : x \rightarrow T(x) + \omega$ the studied application. We wish to maintain the geometrical hypothesis that constrains the position of points in K, and look both for the geometry parameter d, and for the noise parameter, such that the set of points remains (n,ε)-separated by S. To simplify the problem, call y a fixed point chosen in E, with no uncertainty attached to its location so that we can note $[y] = [y,y]$. Now call $[x]$ the location of another point of K, separated from $[y]$ by an unknown distance $[d]$, such that $[x] = [y] + [d]$. Then we apply S to both points and look for the distance of Bowen that

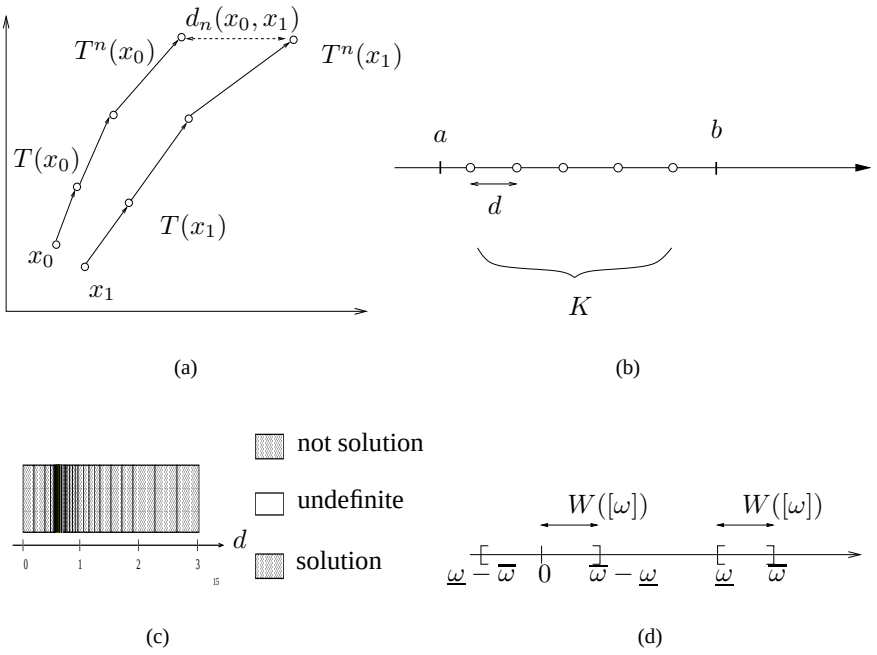


Figure 1. (a) Distance of Bowen between x_0 and x_1. (b) Set K in $E = [a, b]$, with predefi nite geometry parametrized by d. (c) Minimal distance d between points of $K \subset E$ that guarantees their (n, ε)-separation by the application $T : x \to \alpha x$. Here $\alpha = 1.1$, $n = 10$, $\varepsilon = 1.5$. (d) Comparison betwwen $w([\omega] - [\omega])$, and $W([\omega])$.

must satisfy the constraint $d_n([x], [y]) = \mid S^n([x]) - S^n([y]) \mid > \varepsilon$. Elementary calculus shows that:

$$[y] \stackrel{S}{\to} \alpha[y] + [\omega]\ ,\ [x] \stackrel{S}{\to} \alpha([y] + [d]) + [\omega] \tag{5}$$

$$[y] \stackrel{S^n}{\to} \alpha^n[y] + \sum_{i=0}^{n-1} \alpha^i[\omega]\ ,\ [x] \stackrel{S^n}{\to} \alpha^n([y] + [d]) + \sum_{i=0}^{n-1} \alpha^i[\omega] \tag{6}$$

$$d_n([x], [y]) = \Big| \alpha^n[d] + [\sum_{i=0}^{n-1} \alpha^i(\underline{\omega} - \overline{\omega}),\ \sum_{i=0}^{n-1} \alpha^i(\overline{\omega} - \underline{\omega})\ ] \Big| \tag{7}$$

with the hypothesis that $\alpha > 1$. We notice that because of the properties of set computation, $[y]$ vanishes while $\underline{\omega} - \overline{\omega}$ -that characterizes the width of $[\omega]$- appears. We explain this remarking that interval $[\omega] - [\omega]$ is centered around zero, hence the variable that matters is no longer $[\omega]$ but its width $w([\omega] - [\omega])$, as shown by Fig. 1(d).

Fig. 3(a) displays the numerical solution of inequation $d_n([x], [y]) > \varepsilon$ computed by SIVIA, as a function of $[d]$, the distance between $[x]$ and $[y]$, and $W([\omega])$. Note that red areas stand for a guaranteed solution, blue ones show points that are not solution, and yellow denotes points for which nothing can be said. Compared to the previous case when no perturbation was added during the interaction, we find the same qualitative behaviour:

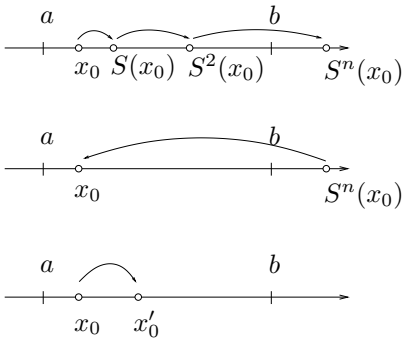


Figure 2. Experimental scheme. *Top:* n steps of a trajectory, starting from x_0. *Center:* back to x_0. *Bottom:* next initial condition x'_0

d must exceed a given value for the separation condition to be verified. However it is worth noting that the minimum required distance d between two consecutive points must increase when the width $W([\omega])$ of additive noise raises. This intuitive result shows that increasing the uncertainty lowers the maximum number of (n, ε)-separated points per volume unit in our simple exmaple.

3. Experimental state-counting

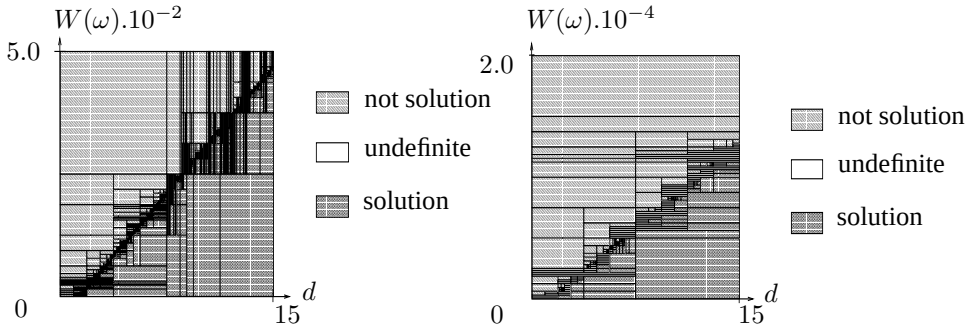
Dealing with the theoretical case in 2, we made a simplifying hypothesis that diserves criticism. Indeed, in order to verify if a set K complies with a given condition, we first have to specify that set of initial conditions K in the state-space. Then the robot must span it and for each initial point, perform a trajectory then jump to the following initial point as shown in Fig. 2. Nevertheless, the robot is not supposed to know how to reach any arbitrary point in the state-space, consequently, K can't be defined before the experiment, and the approximation method presented in 2.4 fails. Instead, as we still want the robot to go along the trajectory driven by S, then to come back near the initial condition and start again with another one, we suggest two hypotheses:

H_1**:** the robot knows T^{-1}, thus it can come back approximately to its starting point.
H_2**:** the robot can jump randomly in the state space and reach a new initial condition.

Our aim here is twofold : first to approximate $S^{-n} \circ S^n([x_0])$, then to inject it in the scheme developped in 2.4 to characterize the necessary jump between two initial conditions belonging to K so that they remain (n, ε)-separated in spite of the uncertainty induced by $S^{-n} \circ S^n$.

3.1. Image of y by $S^{-n} \circ S^n$

In 2.4, we considered a fixed point y, that was written as a trivial interval $[y] = [y, y]$, and the uncertainty concerned the position of the next point $[x]$. In the next section, y won't be considered fixed any longer, because it will be produced by a back-and-forth movement given by $S^{-n} \circ S^n$. Let's focus on the image of a point z with no uncertainty attached to its position, by $S^{-n} \circ S^n$, then we use hypothesis H_1 stated at the beginning of 3, to define S^{-1} . First we write again (6):



(a) $x \to S^n(x)$ (b) $x \to S^{-n} \circ S^n(x)$

Figure 3. Couples $d, w(\omega)$ such that two points of K are (n, ε)-separated by an application. In both cases, $\alpha = 1.1$, $n = 10$, $\varepsilon = 1.5$.

$$[z] \quad \stackrel{S^n}{\to} \quad \alpha^n[z] + \sum_{i=0}^{n-1} \alpha^i[w] \ , \ [z] \stackrel{S^{-n}}{\to} \frac{1}{\alpha^n}[z] + \sum_{i=0}^{n-1} \frac{1}{\alpha^i}[w] \tag{8}$$

$$[z] \stackrel{S^{-n} \circ S^n}{\to} [z] + \sum_{i=0}^{n-1} \alpha^{i-n}[w] + \sum_{i=0}^{n-1} \alpha^{-i}[w] \tag{9}$$

which can't be easily simplified because of particular properties of set computation. We verifiy that although $[z]$ is reduced to a trivial set $[z, z]$, its image by $S^{-n} \circ S^n$ is a non trivial interval in the general case where ω is nontrivial.

3.2. Characterization of the jump $[d]$

Reconsidering the results given in 2.4, we suppress the hypothesis on $[y]$ that makes it a trivial interval. Instead we rewrite (7), replacing $[y]$ by (9), and get:

$$d_n([x], [y]) = \Big| \ \alpha^n[d] + [\sum_{i=0}^{n-1} \beta_{i,n}(\underline{\omega} - \overline{\omega}), \ \sum_{i=0}^{n-1} \beta_{i,n}(\overline{\omega} - \underline{\omega}) \ ] \ \Big| \tag{10}$$

where $\beta_{i,n} = \alpha^i + \alpha^{-i} + \alpha^{i-n}$. Obviously, this constraint $d_n([x], [y]) > \varepsilon$ is more stringent than in (7), because $\beta_{i,n} > \alpha_i$ hence the uncertainty interval is larger. Numerical resolution of the inequation verifies this statement, as shown in Fig. 3(b).

Compared to Fig. 3(b), the solution area (in red) was pushed towards higher values of d. Back and forth movement due to $S^{-n} \circ S^n$ has increased uncertainty, and made more difficult the verification of the (n, ε)-separation constraint.

3.3. Comments

However the purpose of this method must be discussed: it can give predictive insight to the experimenter concerning the limits of the robots, especially concerning its discrimi-

native capabilities. It allows to answer, before the experiment, such questions as "given that level of noise and that application, what will be the minimum distance between two points in phase space such that they can be separated by T". But since the robot has access neither to intervals (but to individual trajectories), nor to interval calculus, it can't determine on-line if two points are (n, ε)-separated.

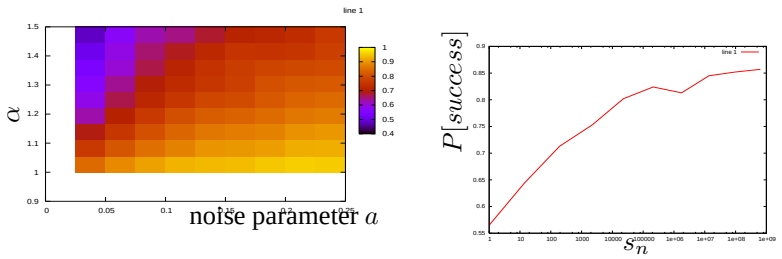
4. Environment recognition

Since the robot knows nothing of the environment except the data it acquires during its interaction, what we call environment recognition is (i) the ability to compare trajectories in the phase space, so as to make some similarity statements about them, (ii) the ability to infer similarity between *locations* in the environment using (i). It is important to remark that the coordinates of a system in an environment may be part of the state space, but that environment and state-space mustn't get confused.

Protocol We keep modelling the interaction of the robot by the application $S : x \rightarrow T(x)+\omega$ where $T(x) = \alpha x$. Now, instead of being modelled as an interval, ω is a random variable that obeys a uniform law $\mathcal{U}(-a, a)$. Thus S is defined on the state-space $X = \mathbb{R}$, which is confused in that particular example with the environment. Then the robot is placed in different initial locations $\{x_i\}_{i\in[1,p]}$ in the state-space, and we let it go along fixed-length trajectories $s_{i\in[1,p]}$, each being defined by $s_i = \{x_i, S(x_i), \ldots, S^n(x_i)\}$. Among the initial conditions $\{x_i\}_{i\in[1,p]}$, one is the origin of two trajectories s_1 and s_2 belonging to $s_{i\in[1,p]}$. Nevertheless s_1 and s_2 are not identical because of the additive random variable. The task for the robot is to pick s_2 among all the others, being given s_1. To do so, it is provided with the distance $< s_i, s_j >= d_{n,S}(x_i, x_j)$ between trajectories. Hence the robot computes all the distance $< s_1, s_k >_{k\in[1,p]}$, and finally picks the trajectory with the shortest distance to s_1. Then, we simply compute the empirical probability of success, as a function of α and of the parameter a.

Results An elementary experiment is the realisation of p trajectories of length n, with different initial conditions, and the pairing of s_1 and s_2 through minimization of $< s_1, s_k >_{k\in[1,p]}$. This elementary experiment is repeated N times so as to compute an empirical probability of success. Then we repeat this with different values of α and a. Results are shown by Fig.4(a). It appears clearly that the probability of success increases when α increases, that is when the exponential behaviour of application $x \rightarrow S^n(x)$ strengthens. On the opposite, when a raises, that is when the range of the uncertainty term driven by the uniform law $\mathcal{U}(-a, a)$ widens, the probability of success decreases.

Comments First, we note that the given result is not predictive, though we emphasized the importance of such a goal in the introduction. Further examination must allow to show before any simulation the expected probability of success. Secondly, suppose we keep constant the parameter of the random variable ω. In 4 we saw that raising α resulted in increasing the probability of success. Now we wish to emphasize the relation between α and $s_n(\varepsilon, E)$, the maximum cardinality of an (n, ε)-separated set $K \in E$. Indeed the relation $s_{n,T}(\varepsilon, E) = [\frac{\alpha^n l}{\varepsilon}]$ holds, in the deterministic case, between α and s_n. Consequently, if we admit temporarily the identification between $s_{n,T}(\varepsilon, E)$ and $s_{n,S}(\varepsilon, E)$, there is a functional relation between s_n and the probability of success for environment recognition, represented by Fig.4(b).



(a) (b)

Figure 4. Probability of success. For each couple of parameter α, a, we run 1000 trajectories of length $n = 50$ steps, in order to compute the empirical probability. (a):Probability of success, as a function of α and the parameter of the random variable ω. (b):Probability of success, as a function of s_n, when the parameter of the random variable ω are fi xed with $a = 0.15$

5. Conclusion, perspectives

In this article we gave a measurable definition of discrimination in robotics, which is a pretty rare topic in that field. We proposed simple mathematical tools to quantify that notion both in a descriptive and predictive manner, and gave an example of calculus. The functional link between our measure and the probability of success of an environment recognition task was underlined. Next we plan first to complexify the example we worked on, then to propose a method that makes the robot able to measure itself the number of separated points in a given volume of the state-space, and to apply it experimentally. For that purpose, we may extend our mathematical scope and involve stochastic tools. Third, we plan to give predictive results of the probability of success of the recognition task. Finally we will look for optimality constraints concerning the whole robot (e.g. architecture, number and technology of sensors and effectors) so as to maximize on-line the density of separated points per volume unit, so as to design feedback laws.

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